

# Marrying level lines for stereo or motion

Nikom SUVONVORN, Samia BOUCHAFA, Bertrand ZAVIDOVIQUE

Institut d'Electronique Fondamentale, Université Paris 11  
Bâtiment 220 - 91405 Orsay Cedex

**Abstract.** Efficient matching methods are crucial in Image Processing. In the present paper we outline a novel algorithm of "stable marriages" that is also fair and globally satisfactory for both populations to be paired. Our applicative examples here being stereo or motion we match primitives based on level lines segments, known for their robustness to contrast changes. They are separately extracted from images, and we draft the corresponding process too. Then for marriages to be organised each primitive needs to be given a preference list sorting potential mates in the antagonist image: parameters of the resemblance founding preferences are explained. Eventually all operators above are embedded within a recursive least squares method and results are shown and compared with a successful Hough based matching that we had used so far.

## 1 Introduction

Efficient matching methods are needed in all areas of Image Processing, ranging from Segmentation - e.g. motion detection or 3D reconstruction from stereo - to actual Pattern Recognition - e.g. model fitting or classification. Efficiency then gets multiple meanings and can address properties as different as easy data extraction and coding format, model simplicity, limited prior assumptions, robustness against ambiguities, conflict freeness etc. not to forget computability. Within that frame, general enough methods are still to be produced. We tested several, from Dynamic Warping on edge or region chain codes [1] for instance to more recently Hough Transform on level lines [2]. In the latter we showed how using n-tuples of carefully coded level line segments, sorted into several sub populations according to the confidence in them, leads to an efficient multi-pass voting process. Efficiency here is in the sense of "fighting ambiguities (e.g. repetitive patterns) thanks to a reinforcement of stronger by weaker features at a limited enough computing expense". In a wider approach to segmentation that aimed at exhibiting nD image features in gathering (n-1)D ones [3] - edges from points of interest, regions from edges etc - an interesting paradigm of optimisation on bi-partite graphs was tested against Bayesian techniques and shown to sustain comparison well with them. It is called the "stable marriage problem", of which the main interest is to guarantee no logical contradiction in the pairing process provided two different sets of items are genuinely distinguished to be cross-matched one-to-one and each item sorts every member of the antagonist set in a so-called "preference list". Efficiency in that case stresses disambiguation

again and we thought to try the method to limit or skip multipass voting in the preceding scheme. The present paper is devoted to preliminary experimentations in that direction for 3D Reconstruction and Motion finding.

It is organized as follows: first, we describe the level-lines' junction extraction resulting into primitives. Second, the preference list construction is explained. Then, the stable marriages algorithm we designed for such matching is outlined. After explaining how outliers are eliminated, we describe shortly the image transformation estimation. Eventually, some results are displayed for comments.

## 2 Image features

The feature type plays a significant role in the choice of a matching strategy. Less reliable features may lead to very complex matching processes. The level-line is chosen here for being a robust/reliable feature, its invariance property towards contrast changes is known [4][5].  $I_{\mathbf{p}}$  being the image intensity at pixel  $\mathbf{p}$ , the *level set*  $N_{\lambda}$  of image  $I$  is made of the pixels which intensities are equal or larger than  $\lambda$ ,  $N_{\lambda} = \{\mathbf{p}/I(\mathbf{p}) \geq \lambda\}$ . Borders of such level sets are called *level lines*  $L_{\lambda}$ . One important property of level lines is that they can overlay but cannot cross. Let  $F_{\lambda}$  be a set of overlaying level lines, called *level line flow*, defined by  $F_{\lambda} = \{L_{\lambda}^i/\lambda \in [u, v]\}$ .  $(v - u)$  is called the *flow extension*  $\mathcal{E}$ . The method proposed in [6][7] extracts the level lines by tracking such flows. In an image, the point where two level line flows merge or split is called a *flow junction*.

The extraction process, under the form of a recursive automaton, performs in 3 steps. First step consists of finding the level line flows at the left-top of each pixel. The following flows are calculated for four directions: top down, right left, bottom up, and left right. Then, flow extension is checked. If there are at least two flows with extensions greater than a threshold, go for the second step, if not pass to the next pixel. The second step consists in validating the flow according to its length despite variations in the flow extension that may make it a subset of the original flow found at first step. So the integral subset flow is looked for continuation in every direction as respecting the following conditions: (1) the predecessor flow must be subset of the successor flow (2) the integral flow does not turn back towards its starting point (3) the integral flow must remain a straight line. Then, we validate flows longer than a threshold. Each validated flow is approximated by a line segment  $S$  characterized by the following 4-vector: starting point  $\mathbf{p}$ , length  $l$ , orientation  $\theta$  and contrast  $(c_l, c_r)$  (average grey level on the left/on the right),  $\vec{S} = [\mathbf{p} \ \theta \ l \ (c_l, c_r)]$ . Its *reliability* is defined as the product  $\mathcal{E} \times l$  of the extension by the length. The last step consists in validating the junction. A junction combines separate flows at a given point –at least two validated flows (line segments)–. As a *primary variable*  $P$ , the junction is characterized by five or seven parameters:  $\Delta\theta$  being the angle between any two line segments,  $\vec{P} = [\mathbf{p} \ \vec{S}_1 \ \vec{S}_2 \ \vec{S}_3 \ \Delta\theta_1 \ \Delta\theta_2 \ \Delta\theta_3]$ .

### 3 Matching candidate

When the primitives have been extracted separately from each image, following the method introduced above, candidates have to be prepared for matching. Each primitive will create a preference list containing its potential mates (the primitives of the other image to be possibly paired). Matching candidates are searched for inside a bounded window the size of which results from a trade off between computation cost and application constraints. The compatibility between primitives is first tested following three constraints on the junction properties : (1) there is at least one common level line between two junctions (2) they must have the same order of area's intensities (3) angles between any two level line flows cannot change by more than 180 degrees. Note that preference lists are thus incomplete (not all primitives in the other image belong to the list) and likely have different number of matching candidates for different primitives. The preferences are set then based on the junction similarity. Two categories are distinguished according to the transformation range in the targetted application.

(1) if the images to match show an important displacement between them, we rather use the junction properties for its similarity  $\mathcal{S}$ : it is a weighted sum of three separate terms: (1) junction reliability  $\mathcal{F}$ , (2) region characteristics around the junction  $\mathcal{R}$ , and (3) geometric invariance  $\mathcal{G}$ .

$$\begin{aligned} \mathcal{S} &= \alpha\mathcal{F} + \gamma\mathcal{R} + \delta\mathcal{G} \\ \mathcal{F} &= \frac{\sum_{i=1}^N \mathcal{F}_i}{N} && \text{with } \mathcal{F}_i \text{ the segment reliability,} \\ &&& N \text{ the segment number} \\ \mathcal{R} &= \frac{\sum_{i=0}^N \|I(\mathcal{R}_i) - I(\mathcal{R}'_i)\|^2}{N} && \text{with } I(\mathcal{R}_i) \text{ the contrast } (c_l, c_r) \\ \mathcal{G} &= \sum_{i=0}^N \left| \frac{l_i}{l_{i+1}} - \frac{l'_i}{l'_{i+1}} \right| + \sum_{i=0}^N |\Delta\theta_i - \Delta\theta'_{i+1}| \end{aligned} \tag{1}$$

(2) if the displacement is small (like in limited motion analysis) a classic similarity can be used, as the correlation or the sum of differences.

### 4 Stables marriages matching

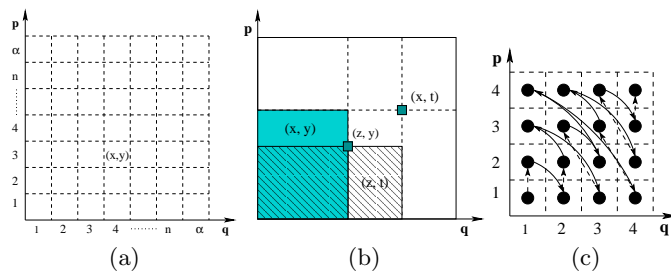
After preselecting matching candidates by creating the preference list for each primitive, an algorithm of stables marriages suitably designed for such problems (see algo.1) will be used for actual matching. Let us first explain the stable marriages paradigm ([8], [9]). In this problem, two finite sub-sets  $M$  and  $W$  of two respective populations, say men and women, have to match. Assume  $n$  is the number of elements,  $M = \{m_1, m_2, \dots, m_n\}$  and  $W = \{w_1, w_2, \dots, w_n\}$ . Each element  $x$  creates its preference list  $l(x)$  i.e. it sorts all members of the opposite sex from most to less preferred. A matching  $\mathcal{M}$  is a one to one correspondence between men and women. If  $(m, w)$  is a matched pair in  $\mathcal{M}$ , we note  $\mathcal{M}(m) = w$  and  $\mathcal{M}(w) = m$  and  $\rho_m$  is the rank of  $m$  in the list of  $w$  (resp.  $\rho_w$  the rank of  $w$  in the list of  $m$ ). Man  $m$  and woman  $w$  form a blocking pair if  $(m, w)$  is not in

$\mathcal{M}$  but  $m$  prefers  $w$  to  $\mathcal{M}(m)$  and  $w$  prefers  $m$  to  $\mathcal{M}(w)$ . If there is no blocking pair, then the matching  $\mathcal{M}$  is stable.

A reliable algorithm of stable marriages in stereo or motion that is global fitting from local attraction, should fulfill three criteria: stability (i.e. no local questioning of more global associations), sex equality (i.e. local/global balance of the resemblance to matching) and global satisfaction (i.e. limited amount of local counter run).

Classic stable marriage algorithms (see [8]) of complexity  $O(n^2)$  guarantee the stability only. The solution can be such that every primitive has a weak fit. The proposed algorithm *Blocked Zigzag (BZ)* (algo.1) meets the three criteria thanks to a novel representation, called *marriage table*, that translates and supplements the preference lists. The *marriage table* is a table with  $(n + 1)$  lines and  $(n + 1)$  columns. Lines (resp. columns) frame the preference orders of men,  $\{1 \dots p \dots N \infty\}$  (resp. women,  $\{1 \dots q \dots N \infty\}$ ). The cell  $(p, q)$  contains pairs  $(m, w)$  such that  $w$  is the  $p^{th}$  choice of  $m$ , and  $m$  is the  $q^{th}$  choice of  $w$ . Cells can thus contain more than one pair or none. The cell  $(p, \infty)$  (resp.  $(\infty, q)$ ) contains the pairs where the woman is the  $p^{th}$  choice of the man (resp the  $q^{th}$  choice of the woman) but the man does not exist in her preference list (resp. the woman is not in his preference list). A key feature of this table in the "complete list" case is that each line contains all men once and each column contains all women once (see figure 1(a)).

Stable matchings are looked for by scanning this latter array and suitable properties of the solution are associated to the type of scan. Indeed, one advantage of the marriage table is that satisfaction, equality of sex and stability show concurrently in the same representation. A solution with maximum global satisfaction would display matched pairs as close around the origin (table bottom-left) as mutual exclusion allows. More generally the table representation is indicative of a result global satisfaction through the lay out of the selected couples. Intuitively the closer to the diagonal the more balanced treatment. Elements of a pair in a cell close to the diagonal are equally satisfied or unsatisfied, depending on the distance to the origin. Stability gets a graphic translation too in the marriage table.



**Fig. 1.** (a) Marriage table : the pair  $(x,y)$ ,  $y$  is the  $3^{th}$  choice of  $x$  and  $x$  is the  $4^{th}$  choice of  $y$ . (b) Blocking situation in marriage table. (c) *BZ* algorithm.

The *BZ* algorithm scans anti-diagonals of the table forward from maximum to minimum global satisfaction but each one is read in swinging from center to sides meaning maximum to minimum sex equality (see figure 1(c)). In each cell, all pairs are accepted for marriage if their components are free. After all cells have been considered, the table is then revisited up to complete removal of blocking situations as follows: potential blocking pairs are matched upon detection (test according to figure 1(b)) while both blocked couples are broken and complementary elements are freed. To overcome cycles in the assignment the number of rescanning is limited to the population size. Scan directions together with questioning all previous marriages on demand guarantees the better at end. However, *BZ* shows an increase in complexity to  $O(n^3)$  due to systematic test added.

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**Algorithm 1:** Blocked zigzag algorithm

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```

begin
  while there is a blocking pair and rescan number < population size do
    foreach anti-diagonal, maximum to minimum global satisfaction do
      foreach diagonal, maximum to minimum sex equality back and forth
      do
        foreach pair (m, w) do
          if m and w are free then
            Marry m with w
        foreach anti-diagonal, maximum to minimum global satisfaction do
          foreach diagonal, maximum to minimum sex equality back and forth
          do
            foreach pair (m, w) do
              if (m, w) is blocking pair then
                Free m and w and their spouse
                Marry m with w
    end

```

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## 5 Eliminating outliers

*BZ* organizes the best correspondence possible for each primitive. Still, when corresponding primitives do not exist in either image, the matching couple cannot be else than a mismatch or a missing match, called *outlier*. We draft here the simple method for eliminating outliers. Considering each couple as an "optical flow" and assuming there are only small displacements locally in images, such optical flow gets same length and direction in average. Whence the algo.2:

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**Algorithm 2:** Outliers elimination algorithm

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```
begin
  foreach optical flow in small window do
    foreach  $x \in \{angle, length\}$  do
      Find neighbors-optical flows-
      Order neighbors by  $x$ 
      Compute  $x$  between any two close flows
      Order  $x$  from min to max
      Find optimal threshold ( $x$  histogram)
      Compute  $x$  in order (current flow, neighbors)
      if  $x > optimal\ threshold$  then
        Delete the current optical flow
    end
  end
end
```

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## 6 Global image transformation

After matching and outliers elimination, couples can be assumed reliable enough to estimating the global transformation between images. We outline here the estimation process in the case of perspective transformation model that requires to estimate 8 parameters. Model is of the form  $\mathbf{y} = \mathbf{X}\mathbf{h}$ , where  $\mathbf{h}$  is the parameters column: it follows 2.

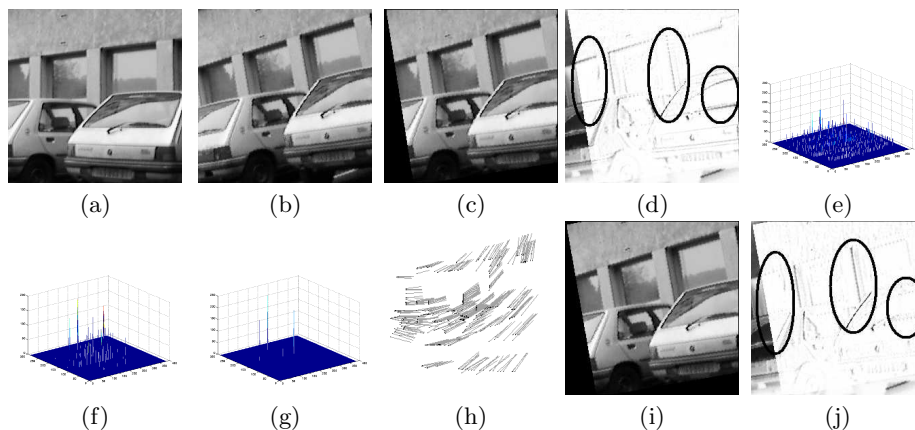
$$\begin{bmatrix} x'_0 \\ y'_0 \\ \vdots \\ y'_N \end{bmatrix} = \begin{bmatrix} x_0 & y_0 & 1 & 0 & 0 & 0 & -x'_0x_0 & -x'_0y_0 \\ 0 & 0 & 0 & x_0 & y_0 & 1 & -y'_0x_0 & -y'_0y_0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & x_N & y_N & 1 & -y'_Nx_N & -y'_Ny_N \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_7 \end{bmatrix} \quad (2)$$

And using the least square recursive method in [10], the transformation parameters are given by  $\mathbf{h} = (\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H\mathbf{y}$ ,  $\mathbf{h} = \mathbf{X}^\#\mathbf{y}$ , hence  $\mathbf{h}_n = \mathbf{Q}_n\mathbf{X}^H\mathbf{y}_n$ ,  $\mathbf{Q}_n = (\mathbf{X}_n^H\mathbf{X}_n)^{-1}$

## 7 Experimental results

In the sequel two series of test results are shown. First one compares the multi pass Hough transform on extracted level-line-segment based primitives, with the stable marriage algorithm run on the same. The difference between original and transformed back images are displayed to that purpose. It is obvious to the naked eye that results are quite comparable, although the BZ Marriages seem to spread errors more all over the picture an in a lesser amount . Each method gets areas where it performs comparatively better – strong primitives orthogonal to the average displacement for Hough and aligned with it for Marriages– . The same phenomenon occurs on all similar images of the type that algorithms were tried on, as long as the transformation to be exhibited is not too complex (limited number of parameters). The present example is extracted from

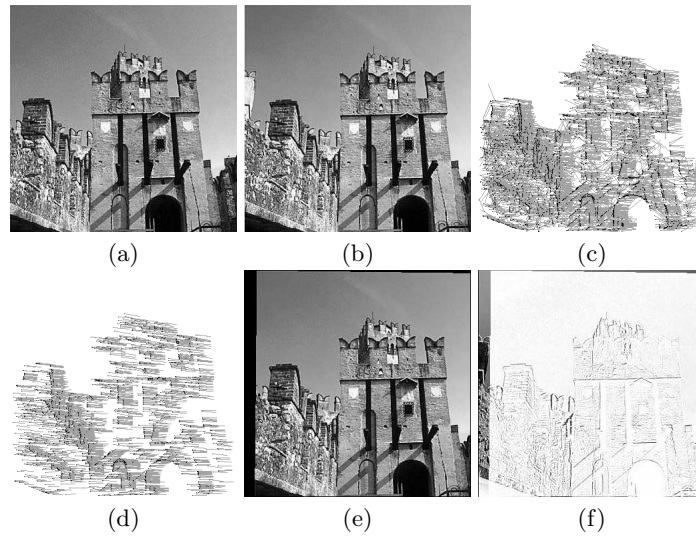
the vision of a car getting out of a parking and involves translations and planar rotations merely. Therefore in a second series of experiments, results of primitive marriages are shown on stereo images extracted from the data base "[http://www.gravitram.com/stereoscopic\\_photography.htm](http://www.gravitram.com/stereoscopic_photography.htm)" of monuments. The algorithm again performs qualitatively well despite projections involved. Good news is that large structures are well distinguished, relatively bad news is that it seems to be to the detriment of more tiny details. That would ask further study of the minimization precision vs. the size of considered primitives, and more generally the impact of the features of selected segments (length, contrast, orientation etc.).



**Fig. 2.** (a)(b) Original images to match, (c) Affine transformation by Hough:  $\alpha = -9.0$ ,  $\psi = 1.0096$ ,  $T_x = 5$  and  $T_y = 3$ , (d) The difference between (b) and (c), (e) The vote space after Hough 1<sup>st</sup> round, (f) 2<sup>nd</sup> round, (g) 3<sup>rd</sup> round, (h) Matching result by *BZ*, (i) Affine transformation by *BZ*:  $\alpha = -9.95$ ,  $\psi = 0.9977$ ,  $T_x = 4.10$  and  $T_y = 6.73$ , (j) The difference between (b) and (i).

## 8 Conclusion

Stable marriages run comparatively well on image couples or sequences segmented into level line based primitives. Our main applications being car driving and experimental physics (electron microscopy, MRI etc.), they involve images with potential high rate of ambiguities. We thus need to compare results with our Hough technique satisfactorily used so far in a more quantitative way and find explanations why one performs better than the other in which cases. That is our next step in the study, as the whole matching process currently under investigation and drafted in the present paper stands, better than many, the growth of the transform parameter number.



**Fig. 3.** (a)(b) The stereo images, (c) Matching results by *BZ*, (d) Result after elimination of outliers, (e) Perspective transformation :  $h_0 = 1.018769$ ,  $h_1 = -0.010705$ ,  $h_2 = 20.975390$ ,  $h_3 = 0.013266$ ,  $h_4 = 0.986765$ ,  $h_5 = 2.239246$ ,  $h_6 = 0.000062$ ,  $h_7 = -0.000035$  and  $h_8 = 1.000000$ , (f) The difference between (b) and (e)

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